Today you will:

- Find maximum and minimum values of quadratic functions
- Practice using English to describe math processes and equations

Core Vocabulary:

- Minimum value
- Maximum value

Minimum and Maximum Values of Quadratic Functions:

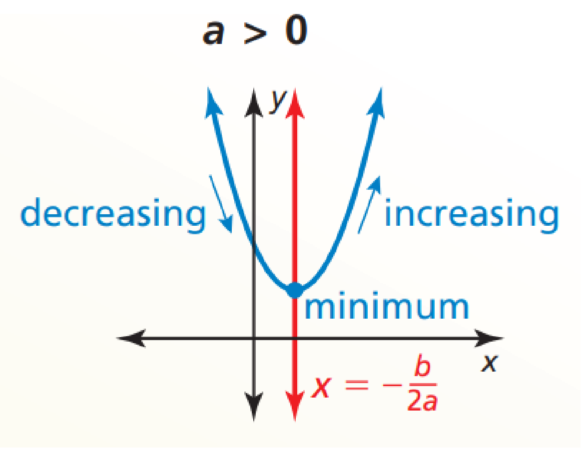
For the quadratic function $f(x) = ax^2 + bx + c$, the *y*-coordinate of the vertex is:

- the ***minimum value*** of the function when $a > 0$
- the ***maximum value*** of the function when $a < 0$

Question:

Why do neither definition include = 0?

Minimum Value



Minimum value: $f\left(-\dfrac{b}{2a}\right)$

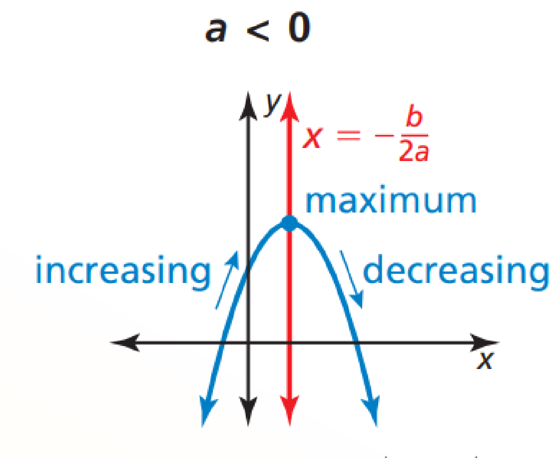
Domain: All real numbers

Range: $y \geq f\left(-\dfrac{b}{2a}\right)$

Decreasing to left of $x = -\dfrac{b}{2a}$

Increasing to right of $x = -\dfrac{b}{2a}$

Maximum Value



Maximum value: $f\left(-\dfrac{b}{2a}\right)$

Domain: All real numbers

Range: $y \leq f\left(-\dfrac{b}{2a}\right)$

Increasing to left of $x = -\dfrac{b}{2a}$

Decreasing to right of $x = -\dfrac{b}{2a}$

Find the minimum value or maximum value of $f(x) = \frac{1}{2}x^2 - 2x - 1$. Describe the domain and range of the function, and where the function is increasing and decreasing.

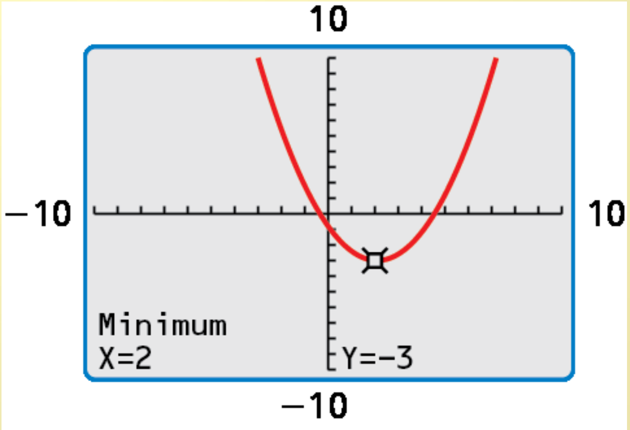
SOLUTION

Identify the coefficients $a = \frac{1}{2}$, $b = -2$, and $c = -1$. Because $a > 0$, the parabola opens up and the function has a minimum value. To find the minimum value, calculate the coordinates of the vertex.

$$x = -\frac{b}{2a} = -\frac{-2}{2\left(\frac{1}{2}\right)} = 2 \quad \Rightarrow \quad f(2) = \frac{1}{2}(2)^2 - 2(2) - 1 = -3$$

The minimum value is -3. So, the domain is all real numbers and the range is $y \geq -3$. The function is decreasing to the left of $x = 2$ and increasing to the right of $x = 2$.

Check



Find the minimum value or maximum value of $f(x) = 4x^2 + 16x - 3$

Describe the domain and range of each function, and where each function is increasing and decreasing.

a = 4
b = 16
c = -3

$$x = -\frac{b}{2a} = -\frac{16}{2 \cdot 4} = -\frac{16}{8} = -2$$

$f(-2) = 4(-2)^2 + 16(-2) - 3 =$ -19

Because $a > 0$ opens up…minimum value

Answer:

- Minimum value is -19
- Domain is all real numbers
- Range is $y \geq -19$
- The function is **decreasing** to the left of $x = -2$
- …and **increasing** to the right of $x = -2$

Find the minimum value or maximum value of $f(x) = -x^2 + 5x + 9$

Describe the domain and range of each function, and where each function is increasing and decreasing.

a = -1
b = 5
c = 9

$$x = -\frac{b}{2a} = -\frac{5}{2\cdot(-1)} = -\frac{5}{-2} = \frac{5}{2} \Rightarrow f\left(\frac{5}{2}\right) = -\left(\frac{5}{2}\right)^2 + 5\left(\frac{5}{2}\right) + 9$$

$$= -\frac{25}{4} + \frac{25}{2} + 9 = -\frac{25}{4} + \frac{50}{4} + \frac{36}{4}$$

$$= \frac{61}{4}$$

Because $a < 0$ opens down…maximum value

Answer:

- Maximum value is $\frac{61}{4}$
- Domain is all real numbers
- Range is $y \geq \frac{61}{4}$
- The function is **increasing** to the left of $x = \frac{5}{2}$
- …and **decreasing** to the right of $x = \frac{5}{2}$

Homework:

- Pg 62, #39-52